

Long Memory in the Realized Volatility of Returns on the Yen/US\$ Exchange Rate during the Three Financial Crises

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Abstract

In this paper, we analyze volatility in high frequency data on returns on the exchange rate for the Japanese Yen against the US dollar during the following economic crises: the Russian financial crisis of 1998 ; the Asian financial crisis of 1997-98 ; and the current global financial crisis, which began in 2008. We in particular analyze the effects of these economic crises on long memory processes in volatility by using the autoregressive fractionally integrated moving average model with an explanatory exogenous variable, which can represent asymmetry in volatility. From this model, we find that there are statistical evidences of long memory and asymmetry in volatility in the returns on the Yen/US\$ exchange rate. We compare the effect of the Russian and Asian financial crises on long memory and find that the former effect on volatility is larger than the latter. Concerning with the current global financial crisis, it is ongoing and hence firmer conclusions on this period await the end of this crisis. However as long as the data up to November, 2008 is concerned we can see that the current global financial crisis has extremely strong effects on the long memory property. Roughly speaking, the size of the shock seems to be associated with the magnitude of the long memory parameter. We may suggest that d could be used as an indicator to evaluate the level of the shocks in economic crises.

Key words: ARFIMAX model, Financial crisis, Exchange rate, High frequency time series, Long memory, Realized volatility

JEL classifications: C22, C52, C53, G14, G15

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1. Introduction

In theoretical and empirical finance studies, fluctuations in financial time series represent volatility, which is measured by the variance (or standard error) of the fluctuations. In classical theoretical research in this field, including research on the Black-Scholes model, the capital asset pricing model (CAPM), and arbitrage pricing theory (APT), volatility is assumed to be unchanged within the particular period under analysis. However, recent studies in empirical finance show that volatility is not constant over time and, hence, models such as the ARCH, GARCH, and Stochastic Volatility (SV) models are used to model variable volatility. Furthermore, authors such as Bekaert and Wu (2000) pointed out that long memory and asymmetric fluctuations are common in the volatility of stock market returns. Subsequently, models designed to represent changeable volatility, such as the autoregressive fractionally integrated moving average (ARFIMA) model, were proposed. This background and the increased availability of high frequency time series stock market data explain why estimating and modeling volatility in financial markets have become important issues. However, there is little research on the asymmetry fluctuations in the exchange rate market returns. McKenzie (2002) pointed out that there is asymmetry in exchange rate volatility. In this paper, we focus on the long memory and asymmetric distribution of volatility, for which the ARFIMA model with exogenous variables (ARFIMAX) is well suited. In our analysis, we use high frequency time series data on the Yen/US\$ exchange rate to calculate realized volatility (RV), which proxies actual (unobservable) volatility. We also analyze the effects of the following economic crises of the last 10-15 years: the Russian financial crisis of 1998 ; the Asian financial crisis of 1997-98 ; and the current global financial crisis, which began in 2008 as a result of subprime loan problems in the USA. Our objective is to determine how these economic crises affect the long memory property in the foreign exchange market. To answer this question, we estimate the long memory parameter of an ARFIMAX model.

In the literature, it is often pointed out that RV follows a long memory process. As explained by Andersen et al. (2003), Giot and Laurent (2004),

Koopman et al. (2005), Ubukata and Watanabe (2005), and Watanabe and Yamaguchi (2005), the ARFIMA model is used to describe long memory in the volatility of RV. Furthermore, the ARFIMA model has been extended to the ARFIMAX model to incorporate the asymmetric property of RV.

Determining the performance of forecast volatility requires knowledge of true volatility, but this is unobservable. True volatility is often proxied by squared stock market returns, as in Watanabe (2000). However, Andersen and Bollerslev (1998) pointed out that, because squared returns contain information on types of variation other than volatility, squared returns tend to underestimate true volatility in ARCH models. They suggested that RV is a more accurate estimate of true volatility.

Moreover, Hansen and Lund (2006) pointed out that it is easy to select a model whose forecast performance is slightly below that of the best forecasting model. They also suggested RV for the proxy variable. Since the recent availability of high frequency financial data and its widespread application in the research field, it is easy to calculate RV from intraday data. Thus, it is possible to examine the performance of the volatility forecast by using RV as the proxy variable. Thus, we follow this method.

However, RV is merely an estimate of actual volatility. Biased estimates of RV produce inaccurate results when RV is used as a proxy variable. Thus, we not only forecast volatility, but also examine its forecasting performance by using Value-at-Risk (VaR). VaR can also be used to check the accuracy of RV as a proxy variable for true volatility. To do this, the models are evaluated by using an $\alpha\%$ VaR threshold. If the assumed model fits the data well, then the percentage (failure rate) that exceeds the $\alpha\%$ VaR threshold should be equal to $\alpha\%$. The proportion of returns that exceed the VaR threshold can be calculated from the estimation.

This paper is organized as follows. In Section 2, we briefly introduce the basic concepts, methods, and models used in the paper, that is, long memory, RV, the ARFIMAX model, and the rolling window. The three subsequent sections are devoted to empirical RV analyses. In Sections 3, 4, and 5, respectively, we conduct RV analyses for the periods of the Russian financial crisis, the Asian financial crisis, and the current global financial crisis.

Concluding remarks are given in Section 6.

2. Models and methods

2.1 Realized Volatility

We assume that logarithm of stock price follows Ito diffusion process

$$d\ln p(s) = \mu(s)dt + \sigma(s)dW(s)$$

where $W(s)$ is Standard Brownian Motion, $\mu(s)$ is the instantaneous rotation (drift), $\sigma(s)$ is the instantaneous volatility. But in this paper $\sigma^2(s)$ is called volatility. The real volatility for t -th day is defined by the integral of instantaneous volatility $\sigma^2(s)$ as follows:

$$IV_t = \int_{t-1}^t \sigma^2(s) ds.$$

IV_t is called Integrated Volatility (IV). Since IV_t is not observable, it has to be estimated based on high frequency data of the intra-day observations of a stock returns $\{r_t, r_{t+1/n}, \dots, r_{t+i/n}, \dots, r_{t+(n-1)/n}\}$ for t -th day, where $t=1, \dots, n-1$. Realized volatility for t -th day is defined by the sum of the square of the intra-day observations as follows:

$$RV_t = \sum_{i=0}^{n-1} r_{t+i/n}^2.$$

When $n \rightarrow \infty$, RV_t converges in probability to IV_t , or RV_t is the consistent estimator of σ_t^2 .

But it is widely recognized in the literature that if n is too large, such as in tick data (ultra-high frequency data), so called micro-structure noise becomes larger and RV is largely biased. To avoid this bias, instead of ultra-high frequency data, five or ten-minute data is used in calculating of RV (see, for example Ait-Sahalia et al. (2005), Bandi and Russell (2004, 2005)).

Therefore, we take five-minute data to calculate RV in this paper.

2.2 ARFIMA Model

If the coefficient of autocorrelation in a time series does not decay for long time, it is said that the series has a long-run dependency, or a long memory. Otherwise, it is said short memory. Formally, short or long memory is defined

as follows:

If the coefficient of autocorrelation of order k , ρ_k , satisfies the condition

$$\sum_{k=1}^{\infty} |\rho(k)| < \infty,$$

then such a time series is called as a short memory process and if it satisfies

$$\sum_{k=1}^{\infty} |\rho(k)| = \infty,$$

then it is called as a long memory process. As long memory process is often observed in many economic time series, several models for describing long memory process have been developed. ARFIMA model is often used to analyze long dependency in volatility of exchange rate. ARFIMA(p,d,q) model is defined as follows:

$$\Phi(L)(1-L)^d(y_t - \mu) = \Theta(L)\varepsilon_t, \varepsilon_t \sim \text{i.i.d.N}(0, \sigma_\varepsilon^2)$$

where d is any real number, L is the lag operator ($L^k y_t = y_{t-k}$), μ is the mean of y_t , and $\Phi(L)$ and $\Theta(L)$ are polynomials in L such that

$$\begin{aligned} \Phi(L) &= 1 - \phi_1 L - \dots - \phi_p L^p, \\ \Theta(L) &= 1 - \theta_1 L - \dots - \theta_q L^q. \end{aligned}$$

We assume that all roots in formulas $\Phi(L)=0$ and $\Theta(L)=0$ are outside of a unit circle. When $d=0$, ARFIMA(p,d,q) is reduced to ARMA(p,q) model and when $d=1$, it is ARIMA($p,1,q$) model. We call d a long memory parameter.

In ARFIMA model, d is a real number and can be negative, but we restrict d to the range of $[0,1]$. According to different values of d , long memory process is categorized as follows:

If $0 < d < 0.5$, it is a stable long memory process.

If $0.5 \leq d < 1$, it is an unstable long memory process.

2.3 ARFIMAX Model

In financial time series, a positive or negative change in a variable may cause a different effect in a stock market. ARFIMA model can not reflect such asymmetry property. To take such asymmetry property into account, ARFIMA model is extended to ARFIMAX(p,d,q) model introduced by Granger and

Joyeux (1980). The general form of ARFIMAX(p, d, q) model is written as

$$(1 - \phi(L))(1 - L)^d(y - X\beta) = (1 + \theta(L))\varepsilon_t, \varepsilon_t \sim \text{i.i.d.N}(0, \sigma_\varepsilon^2)$$

We analyze the long-run dependency in RV in the Yen/US\$ exchange rate by applying this model. In our empirical study of high frequency time series of exchange rate in the section 3.2, we will confine ourselves to the case in which p and q less than or equal to 1 and select p and q by AIC. Finally we take the ARFIMAX(0, d , 1) model as follows:

$$(1 - L)^d\{\ln(RV_t) - \mu_0 - \mu_1|R_{t-1}| - \mu_2 D_{t-1}^-|R_{t-1}|\} = (1 + \theta L)u_t \dots\dots\dots (1)$$

where R_t is the return on the Yen/US\$ exchange rate and $u_t \sim \text{i.i.d.N}(0, \sigma_u^2)$, as in the preceding studies. We call the formula (1) as RV-ARFIMAX model hereafter.

To estimate this model, we adopt Approximate Maximum Likelihood Method proposed by Beran (1995) : $\Theta = (d, \mu_0, \mu_1, \mu_2, \theta)$ is an unknown parameter vector, D_{t-1}^- is a dummy variable defined by

$$D_{t-1}^- = \begin{cases} 0, & R_{t-1} \geq 0 \\ 1, & R_{t-1} < 0 \end{cases}$$

The conditional expectation of $\ln(RV_t)$ for a given RV_{t-1} is written as

$$E[\ln(RV_t)|R_{t-1}] = \begin{cases} \mu_0 + \mu_1|R_{t-1}| & R_{t-1} \geq 0 \\ \mu_0 + (\mu_1 + \mu_2)|R_{t-1}| & R_{t-1} < 0 \end{cases}$$

This formula reflects asymmetry such that if $\mu_2 > 0$ the change in RV after a fall in the exchange rate of ¥/US\$ is larger than that after a rise in it. In other word the appreciation of Yen against US\$ ($R_{t-1} < 0$) on a day $t-1$ leads to higher volatility (RV_t) on the next day t on the average. Therefore, a parameter μ_2 can be used for testing the asymmetric property.

As L is a lag operator, $(1 - L)^d$ can be formally expanded by Taylor expansion at $L=0$:

$$(1 - L)^d = 1 + \sum_{k=1}^{\infty} \frac{d(d-1)\cdots(d-k+1)}{k!} (-L)^k.$$

The long memory parameter d can be used for testing long memory property.

Since we have assumed u_t is i.i.d.N(0, σ_u^2), RV in ARFIMAX model is lognormal distribution.

By solving the equation (1) for $\ln(RV_t)$ we can obtain an approximate formula of RV_t . Then the conditional forecast $\hat{R}\hat{V}_{t|t-1}$ given the information up to $t-1$ can be calculated as follows:

$$\begin{aligned} \hat{R}\hat{V}_{t|t-1} = & \exp[\mu_0 + (\mu_1 + \mu_2 D_{t-1}^-) |R_{t-1}| - \\ & \sum_{k=1}^{\infty} \frac{d(d-1)\cdots(d-k+1)}{k!} (-L)^{k*} \{ \ln(RV_{t-k}) - \mu_0 - (\mu_1 + \mu_2 D_{t-1}^-) |R_{t-k-1}| \} \\ & + \theta \hat{u}_{t-1} + \frac{1}{2} \hat{\sigma}_u^2] \dots\dots\dots (2) \end{aligned}$$

where \hat{u}_t is the residual in the equation (1), $\hat{\sigma}_u^2$ is the variance of the residual. We call (2) as RV-ARFIMAX-mean model in what follows.

$\hat{R}\hat{V}$ is the estimate of the conditional mean of RV. In statistics, the median is more representative than the mean as a location parameter in the lognormal distribution, it may be better to take the median as an estimate of true volatility rather than the mean. The median of the log-normal distribution is expressed by formula (3) which is estimable from data.

$$\begin{aligned} \hat{R}\hat{V}_{median} = & \exp[\mu_0 + (\mu_1 + \mu_2 D_{t-1}^-) |R_{t-1}| - \\ & \sum_{k=1}^{\infty} \frac{d(d-1)\cdots(d-k+1)}{k!} (-L)^{k*} \{ \ln(RV_{t-k}) - \mu_0 - (\mu_1 + \mu_2 D_{t-1}^-) |R_{t-k-1}| \} \\ & + \theta \hat{u}_{t-1}] \dots\dots\dots (3) \end{aligned}$$

We call (3) as RV-ARFIMAX-med model in what follows.

When the true volatility is estimated by the calculated median, it is denoted by $\hat{R}\hat{V}_{med}$. We compare the forecast performance of $\hat{R}\hat{V}$ and $\hat{R}\hat{V}_{med}$ in the next section.

2.4 Rolling window method

In this paper, we estimate $\hat{R}\hat{V}$ and $\hat{R}\hat{V}_{med}$ from ARFIMAX model by rolling window method, then compare $\hat{R}\hat{V}$ and $\hat{R}\hat{V}_{med}$ with the observed RV . Rolling window method is described as follows and illustrated by Figure 1.

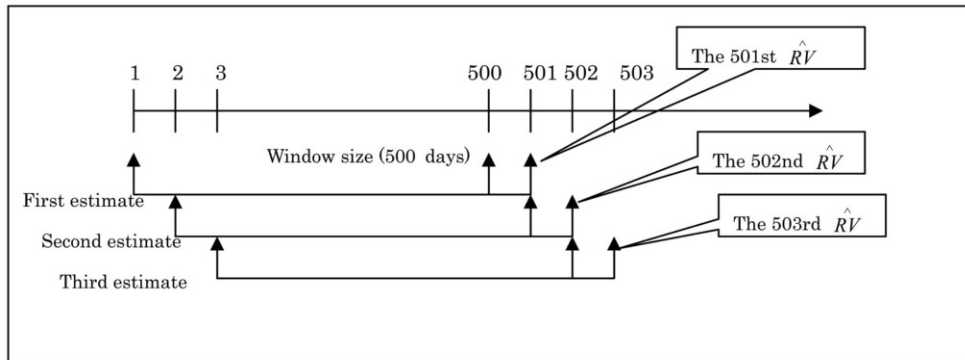


Figure 1 RV forecasted by the rolling window

First, we estimate the unknown parameters in the ARFIMAX model by using 1st- s th observations, and then we forecast the $(s+1)$ st RV by $\hat{RV}_{s+1|s}$. This period with a time span s is called a window and s is called as window size. Secondly, using 2nd- $(s+1)$ st observations we forecast the $(s+2)$ st RV by $\hat{RV}_{s+2|s+1}$, and repeating the procedure of moving the window along the time axis by unit time step and calculating the one step ahead forecast. This method is called as the rolling window method.

3. Empirical analysis: Case of the Russian economic crisis

3.1 The basic statistical characteristics of data and graph

In this section, we use Olsen's high frequency data of Yen/US\$ exchange rate with the time interval of one-minute, from 1991-05-01 to 2006-08-3-1. To avoid micro-structure noise, the time interval of the data is changed from one-minute to five-minute by dividing the original series into subsamples with five-minute time interval, then taking the last data within these subsamples as the observations. The definition of the exchange rate return is $R_t = 100 * \log(P_t / P_{t-1})$. Here, P_t is the mean of the bid and ask in the t -th minute. The daily return is the average of the first and the last values of the exchange rate in a day. The basic statistics of R , RV and $\ln(RV)$ are shown in Table 1, from which we can see the distribution of R is leptokurtosis and heavy tailed compared with the normal distribution. This suggests a possibility that RV is distributed as lognormal distribution. Although it is not definitely

Table 1 Basic Statistical Characteristics

Series:	R	RV	LN - RV
Mean	-0.0031	0.6004	-0.7754
Median	0.0044	0.4297	-0.8446
Maximum	5.0624	34.7100	3.5470
Minimum	-6.3640	0.0389	-3.2458
Std.Dev.	0.6727	0.8041	0.6620
Skewness	-0.4792	21.7044	0.6304
Kurtosis	8.8248	840.0641	4.3181
Jarque-Bera	5749.6680	1.16E+08	548.9371
Probability	0.0000	0.0000	0.0000

true that $\ln(RV)$ is lognormal distribution, but we may assume RV is distributed as lognormal (or $\ln(RV)$ is normal) in what follows.

Figure 2 and Figure 3 show the time series of five-minute Yen/US\$ exchange rate and its return. Figure 4 is RV series. Large jumps in return series, especially an extreme spike in RV series can be observed in time axis 1998, which caused by the Russian financial crisis in 1998. Hedge funds liquidate the open positions of Yen on October 1998 resulting in sudden and sharp rises in exchange rate for Yen (Dollar and Euro depreciation). To deal with such extreme values during the Russian financial crisis we use the following two kinds of data in the following analysis: one is the original time series which includes the period of the Russian financial crisis and the other is processed series which excludes the extreme values. In excluding such extreme

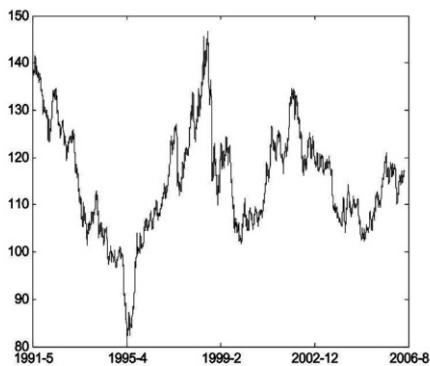


Figure 2 Yen/US\$ Exchange Rate Series
(Period: 19910501–20060831)

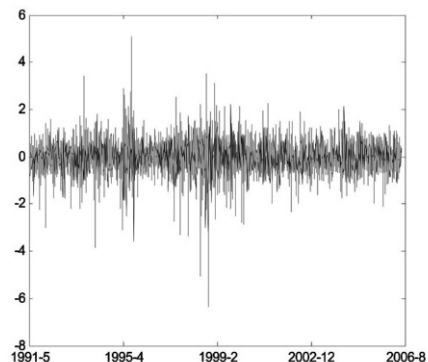


Figure 3 Return Series
(Period: 19910501–20060831)

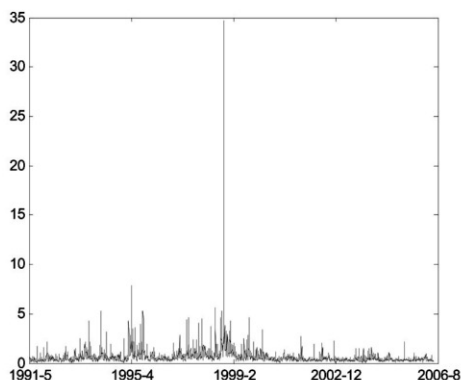


Figure 4 RV Series
(Period: 19910501-20060831)

values, we take two ways: one is to remove three days right in the middle of the Russian financial crisis with the extreme spikes in RV, and the other way is to remove one month before and after the peak of the Russian financial crisis (two months are excluded in total).

3.2 Estimation and forecasting

We estimate the unknown parameters of ARFIMAX(p, d, q) model from the data mentioned above. First of all we selected p and q by AIC and the results are as follows: AIC for ARFIMAX($0, d, 1$) = 1.2098, AIC for ARFIMAX($1, d, 1$) = 1.2184, and AIC for ARFIMAX($1, d, 0$) = 1.8447. Therefore we adopted ARFIMAX($0, d, 1$) model.

In this section we compare the performance of the models considered in this paper. The models are evaluated by $\alpha\%$ VaR-Threshold. If the assumed model is appropriate to the real dataset, the percentage (failure rate) that exceeds $\alpha\%$ VaR threshold should be nearly equal to $\alpha\%$. The $\alpha\%$ VaR threshold at time t is calculated by $c_\alpha \times \sqrt{\hat{RV}_t}$, where c_α is the percentile point of the assumed distribution of return and \hat{RV}_t is calculated from RV-ARFIMAX model. In the empirical application, α equals to 5% and RV follows the lognormal distribution. $c_{5\%} \times \sqrt{\hat{RV}_t}$ is graphed in Figure 10.

Table 2 Estimates of RV-ARFIMAX Model (by rolling window with window size of 500 days)

Period of Crisis	Sample Period	\hat{d}	$\hat{\mu}_0$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\theta}$	$\hat{\sigma}_u^2$	Failure Rate
	Whole period	0.4335	-0.7847	0.0702	0.1074	-0.1099	0.1903	7.99%
Russian Financial Crisis in the narrow sense	Off-peak period	0.4304	-0.782	0.066	0.101	-0.1035	0.1895	7.97%
	Prior period	0.4818	-0.6862	0.085	0.134	-0.1176	0.2093	5.46%
	Posterior period	0.3592	-1.0502	0.0797	0.0378	-0.0741	0.1687	6.37%
Russian Financial Crisis in the wide sense	Off-peak period	0.4221	-0.8006	0.0712	0.0988	-0.0988	0.1894	8.01%
	Prior period	0.4836	-0.6947	0.0869	0.1347	-0.1203	0.2094	5.57%
	Posterior period	0.3578	-1.0537	0.0806	0.0357	-0.073	0.1685	6.37%

Table 2 shows the averages of estimates of RV-ARFIMAX model (1) by rolling window method with the window size of 500 days. It is noted that the failure rate is almost 5 %, and this means that RV-ARFIMAX model is appropriate to the real dataset. If the estimated \hat{d} is greater than 0 and less than 0.5, it means that RV follows a stable long memory process. In the following analysis we divide the sample period as follows: The period of high-tide of the Russian financial crisis, the prior and posterior periods of the high-tide. We also use the following two periods such as “whole period” including all sample periods, and “off peak period” excluding the high-tide period. We capture the Russian financial crisis in two senses: narrow and wide: in the narrow sense Russian financial crisis covers three days of October 7-9, 1998 and in the wide sense two months of September 1- October 30, 1998. Table 2 shows that most estimates \hat{d} 's are around 0.4 in four types of sample periods, i.e., whole sample period including the three periods above, the prior period, the posterior period and the high-tide period. Our estimates $\hat{d} \approx 0.4$ are consistent with reported results in the literature. It shows that \hat{d} is 0.4335 in the whole period, but in the off-peak of crisis in \hat{d} becomes smaller, with the value of 0.4304 in the narrow sense and 0.4221 in the wide sense, which means the Russian financial crisis did affect the realized volatility in exchange market. As described above, μ_2 is a parameter of asymmetry in the sense that if μ_2 is positive, the change in RV after a fall in the exchange rate of ¥/US\$ is larger than that after a rise in the exchange rate. In short positive and negative changes in the exchange rate of ¥/US\$ have asymmetric effects on RV. μ_2 is

Table 3 \hat{d} and its mean \bar{d} of the whole period (rolling window)

Period of Crisis	Sample Period	\bar{d}
	Whole period	0.4335
Russian Financial Crisis in the narrow sense	First period	0.4817
	Middle period	0.4013
	Third period	0.3433
Russian Financial Crisis in the wide sense	First period	0.4835
	Middle period	0.4011
	Third period	0.3426

positive in Table 2, which shows that there is asymmetry in the exchange rate volatility (conditional variance), that is, negative returns lead to higher subsequent volatility than positive returns.

Table 3 shows the estimated \hat{d} by rolling window method and its average \bar{d} . In Table 3 we use the three sample periods, which we call the first, second (or middle), and third period. They are distinguished by the following manner. The first period: all rolling windows in the first period do not include the period of the Russian financial crisis in wide or narrow sense. The second or middle period: all rolling windows in this period do include the period of the crisis in wide or narrow sense. The third period: all rolling windows in this period do not include the period of the Russian financial crisis in wide or narrow sense. We note that \bar{d} is declining through the three periods, from 0.4817 (almost equal to 0.5) to 0.3433 in case of the off-peak period in the narrow sense (excluding the 3 days of the high-tide of the crisis), and from 0.4835 (almost equal to 0.5) to 0.3426 in case of the off-peak period in the wide sense (excluding the 2 months of the crisis). It seems that d is changing from the nearly unstable process in the first period to the stable process in the third. However in the middle of this process, this change in d was risen to bring back to unstable state in the middle of the high-tide of the crisis.

Figures 5-6 show the plot of \hat{d} and its mean \bar{d} calculated from the whole sample period by the rolling window method. Figure 5 is the results of estimation when the Russian financial crisis is treated in the narrow sense in which the high-tide of the crisis is captured by the 3 days of the middle of the

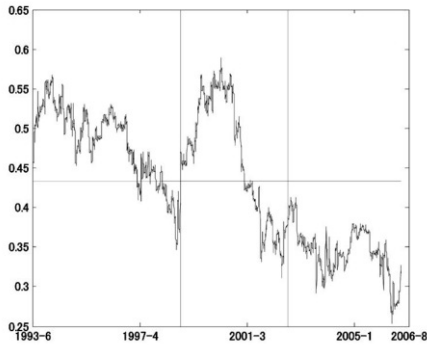


Figure 5 \hat{d} Pass Estimated by Rolling Window and its Average (Off-peak period 3 days) (Period: 19910501-20060831)

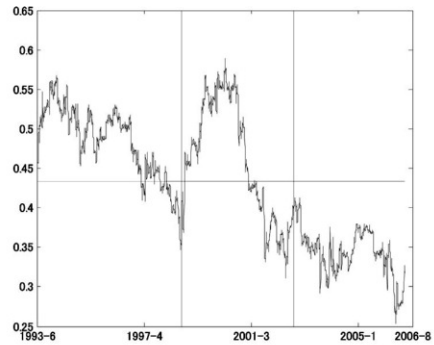


Figure 6 \hat{d} Pass Estimated by Rolling Window and its Average (Off-peak period of the 2 months) (Period: 19910501-20060831)

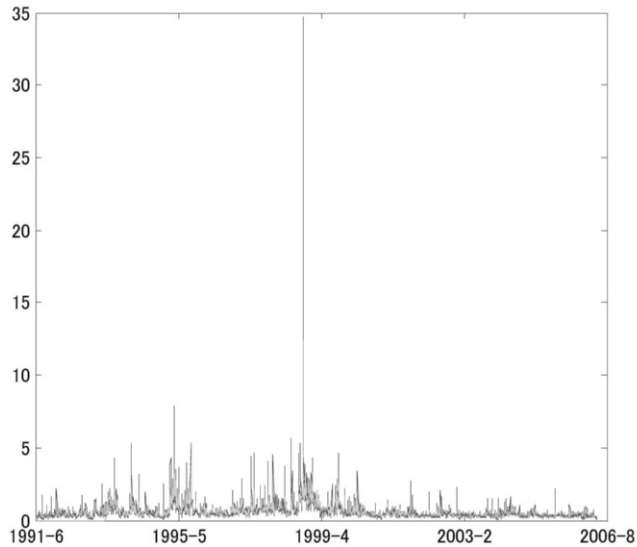


Figure 7 RV and \hat{RV} (Period: 19910501-20060831)

crisis, and the whole period is divided into three sub-periods by vertical lines in the figure. The dotted horizontal line denotes \bar{d} . Figure 6 is the plot of \hat{d} and \bar{d} when the Russian financial crisis is treated in the wide sense in which the high-tide of the crisis is captured by the 2 months of the middle of the crisis.

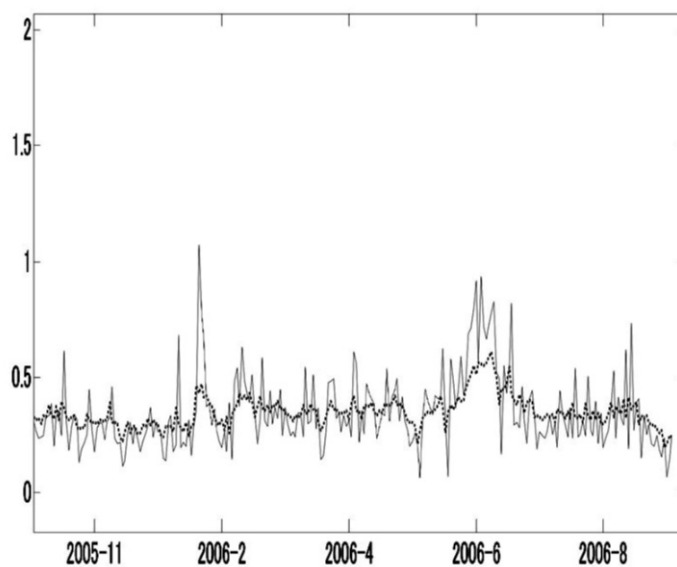


Figure 8 Enlarged View of RV and \hat{RV}
(Period: 19910501-20060831)

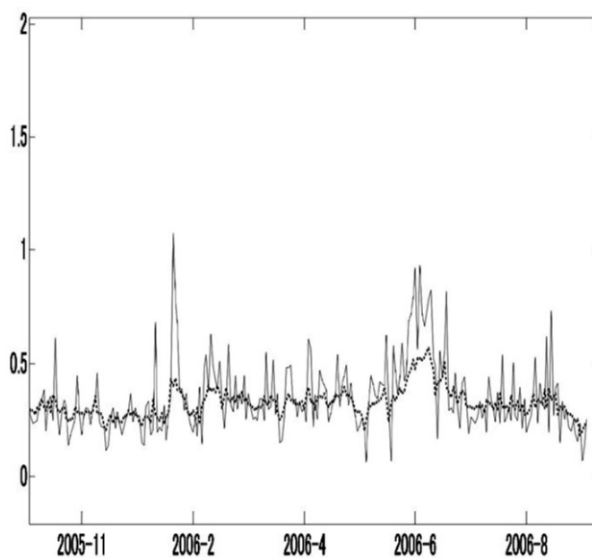


Figure 9 Enlarged View of RV and \hat{RV}_{med}
(Period: 19910501-20060831)

Next, we compare RV with \hat{RV} and \hat{RV}_{med} by figures. Figure 7 is the plot of observed RV and the estimated \hat{RV} that is calculated by RV-ARFIMAX

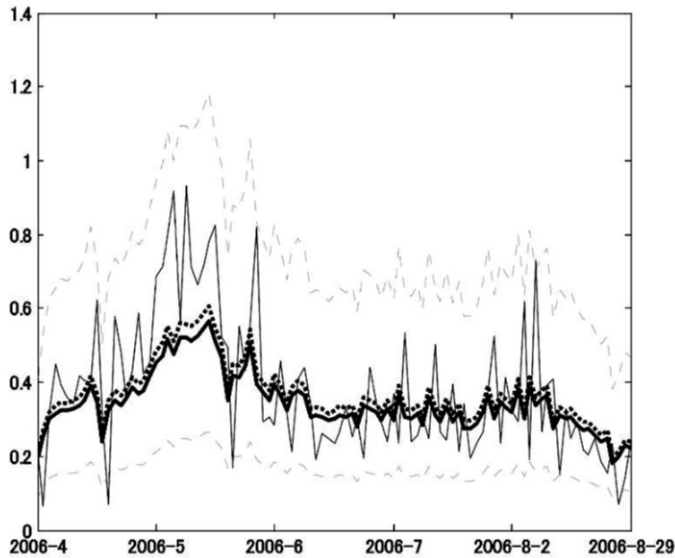


Figure 10 Enlarged View of RV , \hat{RV} , \hat{RV}_{med} and 5% VaR-Threshold (Period: 19910501-20060831)

model in the whole sample. Figure 8 and Figure 9 are enlarged views of RV with \hat{RV} and \hat{RV}_{med} separately. In Figures 8,15, and 23, the solid lines are for RV , and the dotted lines are for \hat{RV} . In figures 9,16, and 24, the solid lines are for RV , the dotted lines are for \hat{RV}_{med} . In Figures 10,17, and 25, the light solid lines are for RV , the bold dotted lines are for \hat{RV} , the bold solid lines are for \hat{RV}_{med} , and the upper and lower dotted lines are for 5% VaR-Threshold. Figure 10 is the enlarged view of $RV, \hat{RV}, \hat{RV}_{med}$ and VaR-Threshold in 5% confidence level of whole sample.

3.3 RV-ARFIMAX model forecast performance

To see the performance of volatility forecast, we use the following measures: RMSE(root means squared error), RMSPE(root means squared percentage error), MAE(mean absolute error) and MAPE(mean absolute percentage error). The performance of the estimated volatility $\hat{\sigma}_{t|t-1}^2$ is evaluated by the distance from the realized volatility RV , which is considered as a proxy variable of the true (unobserved) volatility.

$$\begin{aligned} \text{RMSE} &= \sqrt{\frac{1}{n} \sum_{t=1}^n \left(RV_t - \hat{\sigma}_{t|t-1} \right)^2} \\ \text{RMSPE} &= \sqrt{\frac{1}{n} \sum_{t=1}^n \left(\frac{RV_t - \hat{\sigma}_{t|t-1}^2}{RV_t} \right)^2} \\ \text{MAE} &= \frac{1}{n} \sum_{t=1}^n |RV_t - \hat{\sigma}_{t|t-1}^2| \\ \text{MAPE} &= \frac{1}{n} \sum_{t=1}^n \left| \frac{RV_t - \hat{\sigma}_{t|t-1}^2}{RV_t} \right| \end{aligned}$$

Table 4 Volatility Forecast Performance (Interpolation Method)

	RMSE	RMSPE	MAE	MAPE
RV-ARFIMAX-mean	0.9536	10.4219	0.1749	0.307
RV-ARFIMAX-med	4.3836	3.6088	0.1708	0.2629

where $\hat{\sigma}_{t|t-1}^2$ is the estimate calculated by ARFIMAX model. First, we estimate the unknown parameters of the ARFIMAX model by using the whole sample data, and then calculate the volatility in each time point by interpolation method. Moreover, comparing the model performance when using \hat{RV} and \hat{RV}_{med} as the proxy variable separately, the results are shown in Table 4.

We can see, except RMSE, the performance of RV-ARFIMAX-med is better than that of RV-ARFIMAX-mean.

4. Empirical analysis: Case of Asian financial crisis in 1997–1998

The period of the Asian financial crisis from 1997 to 1998 can be divided into three periods. The first period is from June 1997 to December 1997 when it broke out in Thailand. The second period is from January 1998 to July 1998 when the crisis extends to Indonesia as the Asian financial crisis is deepened. The third period is from August 1998 to the end of the 1998 when the Russian financial crisis broke out. The Asian financial crisis spread out across the regional scope to the global scale, and hence the both economic crises may be treated as one economic crisis. However, we have already analyzed the effects

of the Russian financial crisis in the previous section, therefore, we separate the Asian and Russian financial crises in this section.

Firstly, we graphically examine the general outlook of the Asian financial crisis from June 1997 to July 1998, in which 302 daily observations are contained. In Figure 11 Yen/US\$ exchange rate series is plotted. Return series and its RV series are plotted in Figure 12 and 13 respectively. In Figure 13, we can see that there are many jumps in the RV series and the jump sizes are between 0 and 6. Comparing Figure 13 with Figure 4 we notice that the volatility in RV in the Asian financial crisis is comparatively smaller than that in the Russian financial crisis in which it was between 0 and 35. Therefore, \hat{d} of RV-ARFIMAX model (1) as shown in Table 5 and Table 6 are smaller than \hat{d} in Table 2.

Table 5 shows the ML estimates of RV-ARFIMAX model (1). We note that \hat{d} is 0.3108 that lies between 0 and 0.5, which means that RV follows a stable long memory process during the Asian financial crisis. Table 6 shows the averages of estimates of RV-ARFIMAX model (1) by rolling window method with the window size of 100 days. We note that \hat{d} is 0.2035, which also means RV follows a stable long memory process in the exchange rate volatility. The estimates of μ_2 are positive in Table 5 and Table 6, which

Table 5 ML Estimates of RV-ARFIMAX Model

Date	\hat{d}	$\hat{\mu}_0$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\theta}$	$\hat{\sigma}_u^2$	Failure Rate
19970601-19980731	0.3108	-0.2915	0.0669	0.0287	-0.0126	0.2048	3.49%

Table Table 6 Estimates of RV-ARFIMAX Model (by rolling window, window size = 100 days)

Date	\hat{d}	$\hat{\mu}_0$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\theta}$	$\hat{\sigma}_u^2$	Failure Rate
19970601-19980731	0.2035	-0.2797	0.0404	0.0891	-0.1267	0.1953	3.49%

Table 7 Volatility Forecast Performance (Interpolation Method)

	RMSE	RMSPE	MAE	MAPE
RV-ARFIMAX-mean	0.5488	3.9639	0.5148	0.5937
RV-ARFIMAX-med	6.4165	5.1774	0.5105	0.5617

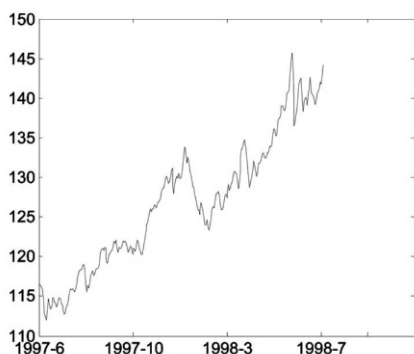


Figure 11 Yen/US\$ Exchange Rate Series
(Asian Financial Crisis: 199706–199807)

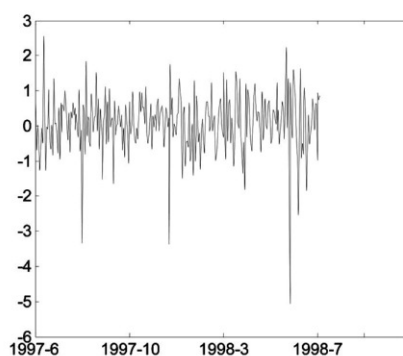


Figure 12 Return Series
(Asian Financial Crisis: 199706–199807)

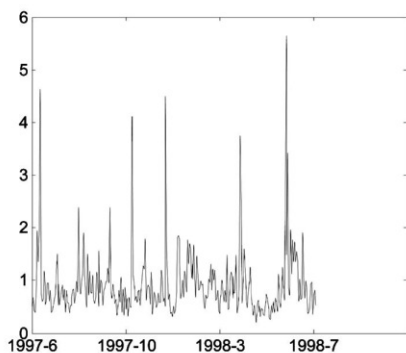


Figure 13 RV Series
(Asian Financial Crisis: 199706–199807)

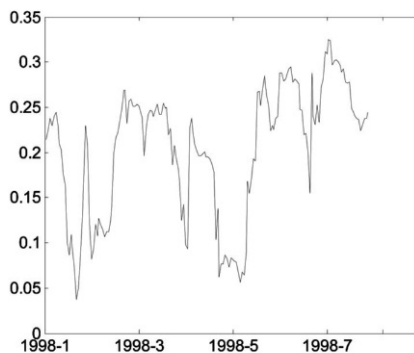


Figure 14 \hat{d} Pass Estimated by rolling window
(Asian Financial Crisis: 199706–199807)

mean that asymmetry was detected for the exchange rate. Table 7 shows the performance of RV-ARFIMAX-med and RV-ARFIMAX-mean models, from which we can see the performance of RV-ARFIMAX-med is better than RV-ARFIMAX-mean model for using \hat{RV}_{med} as the proxy variable evaluated by MAE and MAPE indicators, but worse in respect of RMSE and RMSPE.

Figure 14 shows the plot of \hat{d} estimated by rolling window. Figure 15 is the plots of observed RV and estimated \hat{RV} calculated by RV-ARFIMAX

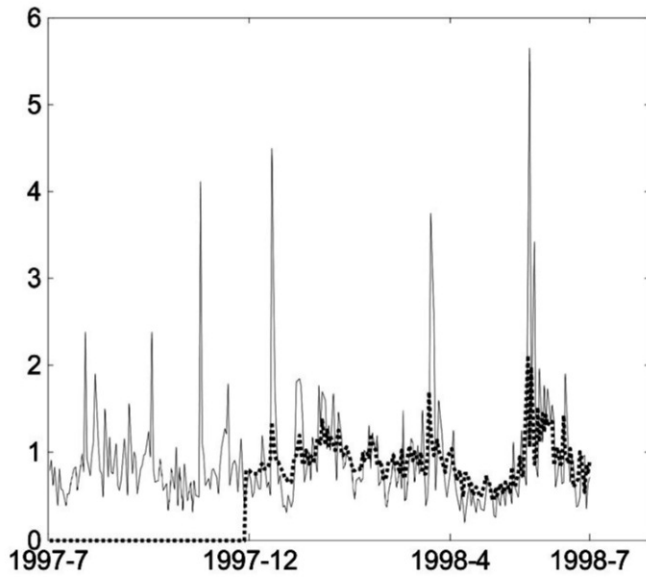


Figure 15 RV and $\hat{R}V$
(Asian Financial Crisis: 199706–199807)

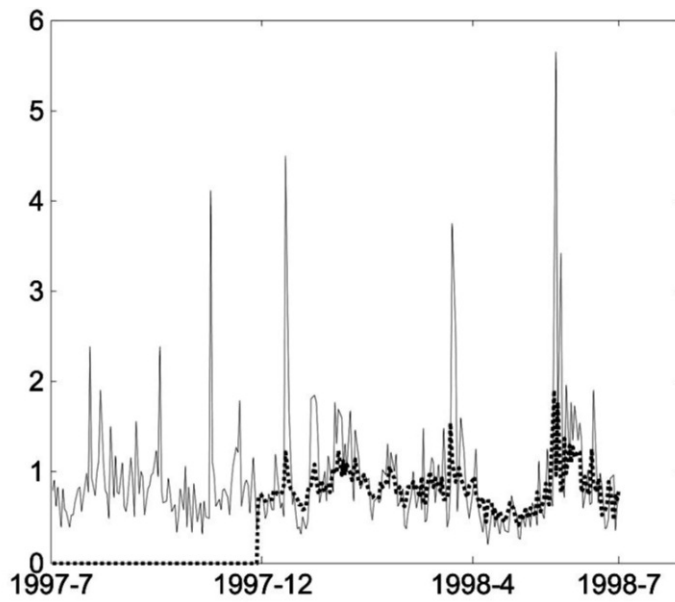


Figure 16 RV and $\hat{R}V_{med}$
(Asian Financial Crisis: 199706–199807)

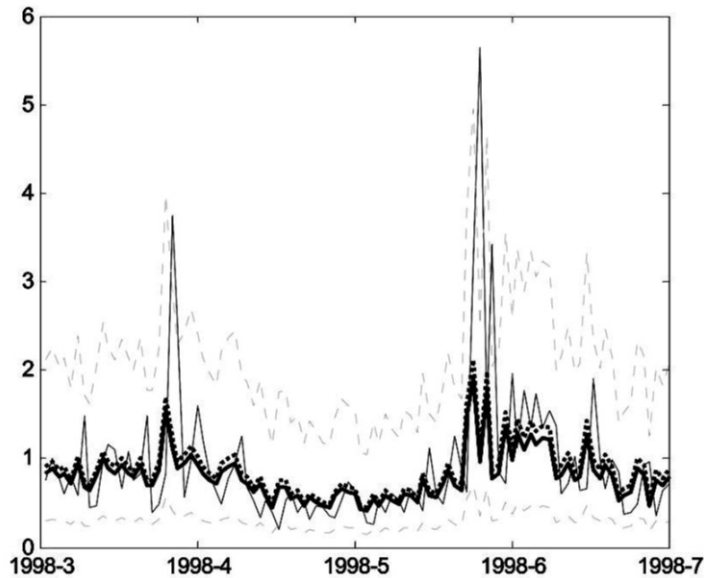


Figure 17 Enlarged View of RV , \hat{RV} , \hat{RV}_{med} and 5% VaR-Threshold
(Asian Financial Crisis: 199706–199807)

model. Figure 16 shows the plots of RV and \hat{RV}_{med} . Figure 17 shows the enlarged views of RV , \hat{RV} , \hat{RV}_{med} and 5% VaR-Threshold. From these graphs, we can see the ARFIMAX model can depict the behavior of volatility in RV very well.

As is seen in Tables 5-6, and Figure 14, the long memory parameter d seems stable and unaffected through the Asian financial crisis. However the estimated parameters d and μ_2 show that there exist the long memory and asymmetry in RV. From Figures 15-17 we can see that the performance of ARFIMAX model is quite good.

5. Empirical analysis: Global Financial Crisis in 2008

To see the effect of the US financial crisis, we add the Olsen's high frequency exchange rate of Yen/US\$ data from 2006-09-01 to 2008-11-30 into the data series. To avoid mixing up the effects of the Asian and Russian crises, we confine ourselves to the period between 2002-01-01 and 2008-11-30, which contains 4544 observations in total. We choose this period because the global

economy was going down after the Asian financial crisis until the economic recovery in 2002. In this section, the window size is set by 100 days, because the sample size is comparatively small when the sample period is divided into sub-samples. Figure 18 and 19 are the plot of the Yen/US\$ exchange rate, Figure 19 is the plot of return and Figure 20 is the plot of RV for the period from 2002-01-01 to 2008-11-30. We can observe sharp rises at the end of the RV series. It is obviously caused by the latest financial crisis caused by the sub-prime loan problem in USA.

Taking into account that the USA financial crisis caused by sub-prime loan problem broke out in August 2007, we divide the data into the following three periods to analyze the effect of the current crisis in this section. Period 1 is the pre-financial crisis period of 20020101-20080731. Period 2 is the mid-financial Crisis period of 20070801-20081130. Period 3 is the whole sample period of 20020101-20081130.

Table 8 shows the ML estimates of RV-ARFIMAX model (1). In this

Table 8 ML Estimates of RV-ARFIMAX Model

Period	Date	$\hat{\alpha}$	$\hat{\mu}_0$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\theta}$	$\hat{\sigma}_u^2$	Failure Rate
Pre Finacial Crisis	20020101- 20070731	0.3945	-1.1641	0.0069	0.0518	-0.0876	0.1688	5.37%
Mid Finacial Crisis	20070801- 20081130	0.5820	-0.4110	0.0122	0.1243	-0.0281	0.2294	5.96%
Whole sample period	20020101- 20081130	0.4973	-1.0202	0.0261	0.1316	-0.1386	0.2026	4.61%

Table 9 Estimates of RV-ARFIMAX Model (by rolling window, window size = 100 days)

Period	Date	$\hat{\alpha}$	$\hat{\mu}_0$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\theta}$	$\hat{\sigma}_u^2$	Failure Rate
Per Finacial Crisis	20020101- 20070731	0.3138	-1.1945	0.0088	0.0345	-0.0598	0.1523	5.98%
Mid Finacial Crisis	20070801- 20081130	0.4964	-0.3842	0.0249	0.1664	-0.0045	0.2101	6.42%
Whole sample period	20020101- 20081130	0.3430	-1.0863	0.0243	0.0781	-0.0587	0.1773	3.35%

table \hat{d} is the estimated parameter of long memory d in Period 1 is 0.3945, which indicates that RV follows a stable long memory process before the current crisis. On the other hand \hat{d} in Period 2 is 0.5280 (> 0.5), which means RV follows an unstable long memory process during the period of the current crisis, and \hat{d} in Period 3 is 0.4973, almost equal to 0.5, which means RV follows a nearly unstable long memory process when the financial crisis is included. The results in Table 8 show that the failure rate in each period is around 5%, which means the RV-ARFIMAX model is appropriate to the real dataset. The parameter of asymmetry μ_2 is positive here, which means that there is asymmetry in the exchange rate volatility.

Table 9 shows the estimates of RV-ARFIMAX model (1) by the rolling window method with the window size of 100 days. We can see the differences in the estimates of d by using the rolling window method in Table 9 and those by not using it in Table 8 for the same data set applied. The former estimates are relatively smaller than those by the latter method because the rolling window method may smooth the fluctuation in time series and may have a similar effect caused by the moving average.

\hat{d} in the Period 1 is 0.3138 which means that RV follows a stable long memory process in the exchange rate volatility before the financial crisis. It is consistent with the result in Table 8. \hat{d} in the Period 2 is 0.4964, almost equal to 0.5, which means that RV follows a nearly unstable long memory process when the current crisis is included. It is consistent to the result in Table 8. \hat{d} in the Period 3 is 0.3430, which means that RV follows a stable long memory process in the exchange rate volatility between 2002-01-01 and 2008-11-30. It is not consistent with the result in Table 8 but seems natural because the rolling window method smooths the fluctuation in a time series. μ_2 are positive in the three periods, which means that negative returns lead to higher

Table 10 Volatility Forecast Performance (Interpolation Method)

	RMSE	RMSPE	MAE	MAPE
RV-ARFIMAX-mean	1.7551	5.2832	0.1526	0.3687
RV-ARFIMAX-med	3.5221	0.7266	0.1526	0.3266

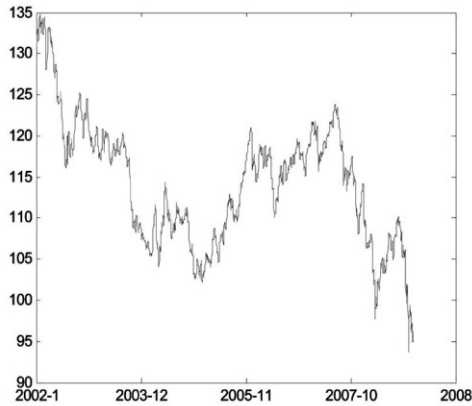


Figure 18 Yen/US\$ Exchange Rate Series (20020101–20081130)

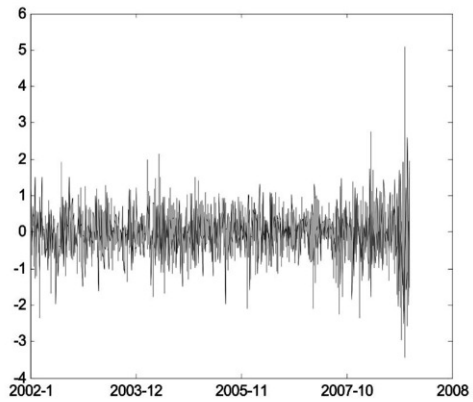


Figure 19 Return Series (20020101–20081130)

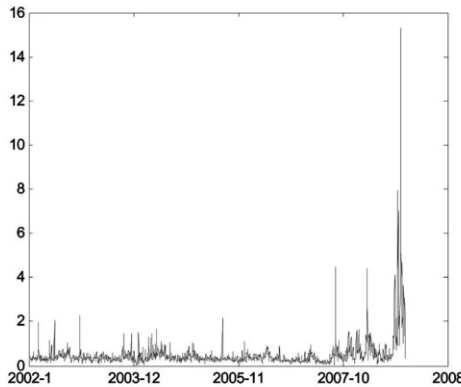


Figure 20 RV Series (20020101–20081130)

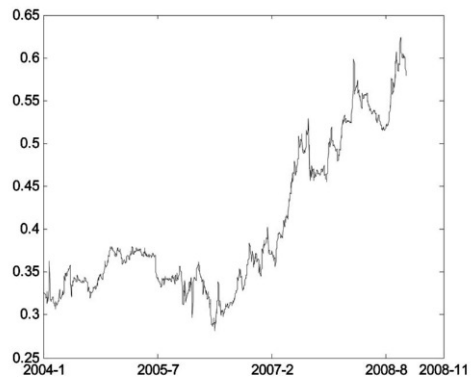


Figure 21 \hat{d} Pass Estimated by Rolling Window (20040116–20081128)

subsequent volatility than positive returns, that is, there exists the asymmetry in the exchange rate volatility. Table 10 shows the performance of RV-ARFIMAX-med and RV-ARFIMAX-mean model, from which we can see the performance of the former is better than the latter except in RMSE.

In Figure 21, the pass of the estimated \hat{d} by rolling window is plotted. In Figure 22 RV and \hat{RV} are plotted, Figure 23 and Figure 24 are the enlarged views of RV with \hat{RV} and \hat{RV}_{med} separately. Figure 25 is the enlarged views of RV , \hat{RV} , \hat{RV}_{med} and 5% VaR-Threshold. From these graphs, we can see the ARFIMAX model can depict the volatility of RV very well. We also notice

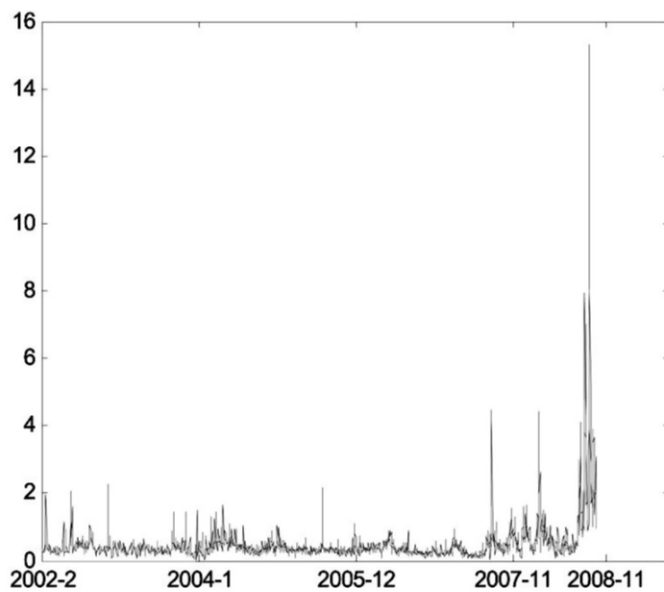


Figure 22 RV and \hat{RV}
(20020101-20081130)

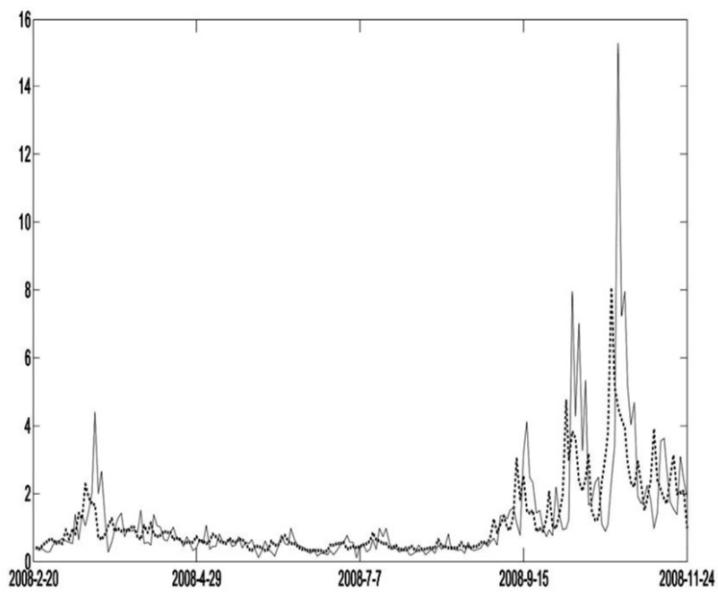


Figure 23 Enlarged Views of RV and \hat{RV}
(20080220-20081124)

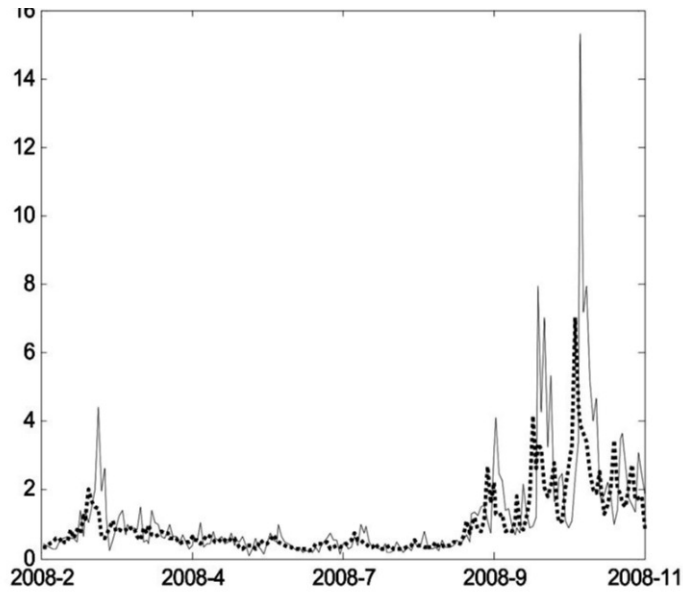


Figure 24 Enlarged Views of RV and $\hat{R}V_{med}$
(20080220–20081124)

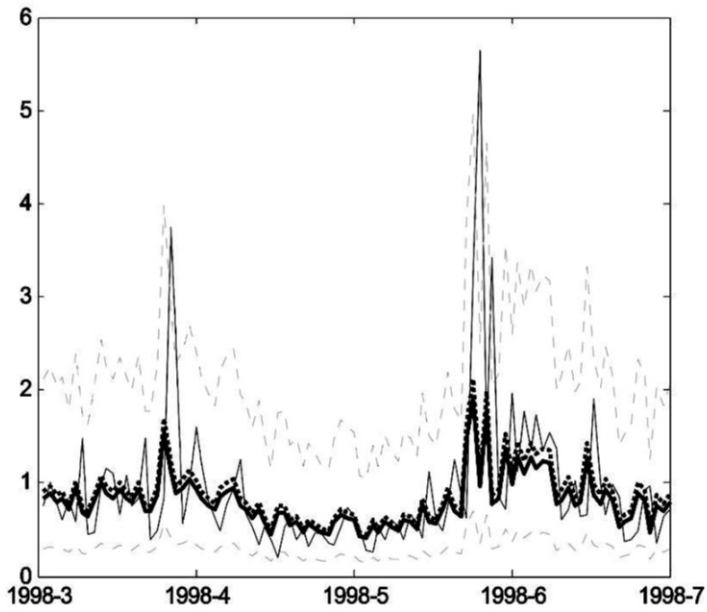


Figure 25 Enlarged Views of RV , $\hat{R}V$, $\hat{R}V_{med}$ and 5% VaR-Threshold
(20080708–20081124)

that d is rapidly increasing through the current global financial crisis as is seen in Figure 21.

Before the current financial crisis \hat{d} is less than 0.5 both in Tables 8 and 9, which means that RV is stationary process. However during the period of the current financial crisis, \hat{d} is greater than 0.5 in Table 8 and just below 0.5 in Table 9. This means that RV is almost non-stationary in this period. From this we can easily infer that the current financial crisis makes the stable RV process in the foreign exchange market unstable. As the current financial crisis is still continued, it may be too early to assess thoroughly the effect of the current financial crisis on the long memory parameter d , but we can see the sharp impacts of the current financial crisis on the realized volatility of exchange rate as is seen in Figures 18-25. In the future, we plan further analysis of the effect of the current global financial crisis.

6. Concluding Remarks

In this paper, using Olsen's one-minute high frequency data on the Yen/US\$ exchange rate, we modeled fluctuations represented by realized volatility (RV) by estimating the parameters of an autoregressive fractionally integrated moving average with an explanation (exogenous variables) (ARFIMAX) model. We used the maximum likelihood and rolling window methods to estimate the model over three periods of economic crisis; that is, the Russian, Asian, and global financial crises. We found that, for those periods, RV exhibited long memory and the property of asymmetry. We also compared the impact of the three economic crises on the long memory parameter d of the ARFIMAX model. The size of the shock tends to be associated with the magnitude of the long memory parameter d value. The economic crisis can affect the long memory property in the foreign exchange rate market as shown in the estimates of d .

Our estimate of d declined sharply during the period of the Russian financial crisis. This suggests that, taking the crisis period as the boundary, the RV series changed from being an unstable long memory process to being stable thereafter. More precisely, in the middle of this process, the decline in d was

arrested by the crisis and then rose to become unstable during the peak of the crisis, after which d resumed its decline.

We also found statistical evidence of asymmetry in the RV series for the exchange rate in the form of a positive estimate of the asymmetry parameter μ_2 in the ARFIMAX model. If μ_2 is positive, the change in RV after a fall in the exchange rate of ¥/US\$ is larger than that after a rise in it. In short positive and negative changes in the exchange rate of ¥/US\$ have asymmetric effects on RV.

We also compared the performance of our estimate of volatility from an RV-ARFIMAX-mean $(0, d, 1)$ model with that from an RV-ARFIMAX-med $(0, d, 1)$ model. The former uses the mean of RV as its proxy variable whereas the latter uses the median. For this comparison, we used RV as a benchmark. Performance was measured by using the root mean squared error, the root mean squared percentage error, the mean absolute error, and the mean absolute percentage error. We found that the RV-ARFIMAX-med model outperformed the RV-ARFIMAX-mean model. This reflects the well-known fact that, in the lognormal distribution, the median is a better representation of the distribution than the mean.

Although the effect of the Asian financial crisis on long memory is less clear than is that of the Russian financial crisis, its effects are discernable from our estimate of d . Tables 5 and 6 and Figure 14 show that the long memory estimate of d is stable and seemingly unaffected by the Asian financial crisis. However, our estimates of d and μ_2 suggest long memory and asymmetry in RV. Figures 15-17 indicate that the ARFIMAX model performs well.

We also analyzed the effect of the current global financial crisis on long memory. Because the current crisis is ongoing and because time series data on a sufficiently long period are not yet available, conclusions about the effect on long memory of this crisis are inevitably premature. However, the analysis reported in Section 5 may constitute an informative initial assessment that provides some perspective on the current global economic crisis. Figures 20, 22, and 23 illustrate a dramatic change in RV. Figure 21 indicates that d increased rapidly during the current global financial crisis.

According to the magnitude of d value, the degree of the financial crisis is

ordered as follows: the current global financial crisis, the Russian crisis and Asian financial crisis. The long memory series moved from a stable process in the pre-crisis periods to an unstable process during the crises. In the future, we plan further analysis of the effect of the current global financial crisis.

Acknowledgment

This research was financially supported in part by Grant-in-Aid for Scientific Research (Grant No. 18330041) of the Japan Society for the Promotion of Science and by Grant for Research Project of Hiroshima University of Economics. This paper is an extension of a part of the doctoral dissertation “Analysis of Financial Time Series by Econometric Models” by Lu Xinhong (2007) and also a part of “Modeling RV in the Exchange Rate” by Lu Xinhong, Tee Kian Heng, Ken-ichi Kawai, and Koichi Maekawa, presented at the annual meeting of Japan Statistical Association held in Kobe University in 2007. We are grateful to Tee Kian Heng, Ken-ichi Kawai and Toshiaki Watanabe for their valuable comments and discussion.

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